Supplementary Information

1 Supplementary methods

Experimental details

The precise definition of climate sensitivity (S) we used in the survey is as follows:

> "An increased concentration of carbon dioxide (CO2) has a **direct effect** on the Earth's energy balance, purely due to the change it represents in the chemical composition of the atmosphere.
>
> Increased CO2 concentrations also have **indirect effects** on the energy balance, through the feedbacks in the climate system.
>
> Consider both direct and indirect effects. **Climate sensitivity** (S) is defined as follows:
>
> Suppose that the atmospheric concentration of carbon dioxide (CO2) is instantaneously increased from its preindustrial level to twice its preindustrial level, and maintained at this new level forever.
>
> We define the climate sensitivity (S) as the resulting increase (in degrees Celsius) in global mean surface air temperature in equilibrium, accounting for both **direct** and **indirect** effects."

The survey first elicited the 5th, 50th, and 95th percentiles of each scientist's hypothesized CDF for S using standard direct probabilistic elicitation techniques [1]. For $p \in \{0.05, 0.5, 0.95\}$, denote the elicited 100pth percentile by S_p. Now define the following urns:

- E: The Ellsberg Urn, which contains 100 balls, each either red or blue, with an *unknown* proportion of each colour.
- O_p: The Objective Urns, also containing 100 balls, which are known to contain $100(1-p)$ red balls, and $100p$ blue balls, for $p \in \{0.05, 0.5, 0.95\}$.

Also, denote the hypothetically known true value of climate sensitivity by S. The following bets are defined:

- $R(E)$: pays out \$50 if a red ball is drawn from Urn E, and zero otherwise.
- $B(E)$: pays out \$50 if a blue ball is drawn from Urn E, and zero otherwise.
- $R(O_{0.5})$: pays out \$50 if a red ball is drawn from Urn $O_{0.5}$, and zero otherwise.
- $B(O_{0.5})$: pays out \$50 if a blue ball is drawn from Urn $O_{0.5}$, and zero otherwise.

· $S^+(0.5)$: pays out \$50 if $S > S_{0.5}$, and zero otherwise.

· $S^-(0.5)$: pays out \$50 if $S < S_{0.5}$, and zero otherwise.

And for $p \in \{0.05, 0.95\}$:

· $R(O_p)$: pays out \$1000p if a red ball is drawn from Urn O_p, and zero otherwise.

· $B(O_p)$: pays out \$1000$(1-p)$ if a blue ball is drawn from Urn O_p, and zero otherwise.

· $S^+(p)$: pays out \$1000p if $S > S_p$, and zero otherwise.

· $S^-(p)$: pays out \$1000$(1-p)$ if $S < S_p$, and zero otherwise.

Experts were asked to express strict preference or indifference for each of the following pairs of bets:

· Ellsberg Problem: $EP_1 = [R(O_{0.5}), B(O_{0.5})], EP_2 = [R(E), B(E)], EP_3 = [R(O_{0.5}), R(E)]$.

· Climate Problem: $CP_1(p) = [R(O_p), B(O_p)], CP_2(p) = [S^+(p), S^-(p)]$,
$CP_3(p) \in \{[R(O_p), S^+(p)], [R(O_p), S^-(p)], [B(O_p), S^+(p)], [B(O_p), S^-(p)]\}$

Note that there is a different Climate Problem at each of the three values for p, and $CP_3(p)$ is determined by the expert's choices in $CP_1(p)$ and $CP_2(p)$ in a manner to be described below.

As Ellsberg showed, many people express indifference on the pairs EP_1 and EP_2, but have a strict preference for $R(O_{0.5})$ in EP_3, preferring to bet on a known risk rather than an unknown risk. These preferences violate the axioms of SEU, and indicate an aversion to the ambiguity in Urn E. If experts' knowledge about S is ambiguous, and they are also averse to ambiguity (as revealed by their choices on the Ellsberg Problem), they may express this in bets on the value of S. The Climate Problem allows us to detect ambiguous knowledge about S. In analogy with the Ellsberg Problem, scientists were first presented with choices between bets with known probabilities ($CP_1(p)$), and on the value of S ($CP_2(p)$). The third pair of bets $CP_3(p)$ depends on their responses in the previous two choices. It consists of one bet from pair $CP_1(p)$ and one from $CP_2(p)$, selected so that experts have an opportunity to express ambiguity aversion. The next section describes how we construct the third betting pair $CP_3(p)$, and how violations of SEU are inferred from observed choices in both the Ellsberg and Climate Problems.

Identifying violations of Subjective Expected Utility

The Ellsberg Problem and the Climate Problems correspond to two variants of Ellsberg's 2-urn choice experiment: the classic version where all bets have equal payoffs (represented in Figure 1 in the paper), and a modified version where bets have unequal payoffs. The classic version corresponds to the Climate Problem at $p = 0.5$, and of course the Ellsberg Problem itself. The version with unequal payoffs corresponds to the Climate Problem at $p = 0.05$ and $p = 0.95$. In what follows, we explain the inferences that can be drawn from observed choices between pairs of bets in each case.

Classic Ellsberg Problem: In order to describe inference in this case we will focus on an example for the Climate Problem at $p = 0.5$, since this will allow us to demonstrate how we construct an appropriate third betting pair CP_3. The logic we describe applies equally well to the Ellsberg Problem however.

For the Climate Problem at $p = 0.5$ we have one urn with known proportions of red and blue balls (Urn $O_{0.5}$), and bets on whether $S > S_{0.5}$. Experts are initially asked to choose between the pairs of bets $CP_1(0.5)$ and $CP_2(0.5)$. Let R (B) denote the outcome in which a red (blue) ball is drawn from Urn $O_{0.5}$, and + (-) denote the outcome that the true value $S > S_{0.5}$ ($S < S_{0.5}$). The space of possible outcomes in these two bets is $(1) = (R, +), (2) = (R, -), (3) = (B, +), (4) = (B, -)$. The payoffs for the bets experts were asked to consider, $R(O_{0.5}), B(O_{0.5}), S^+(0.5), S^-(0.5)$, are depicted in Table 1. Bet $R(O_{0.5})$, for instance, yields a payoff of x only when a red ball is drawn from Urn $O_{0.5}$, irrespective of the value of S.

Table 1: State space for the Climate Problem at the 50th percentile

	(1) $(R, +)$	(2) $(R, -)$	(3) $(B, +)$	(4) $(B, -)$
$R(O_{0.5})$	x	x	0	0
$B(O_{0.5})$	0	0	x	x
$S^+(0.5)$	x	0	x	0
$S^-(0.5)$	0	x	0	x

Suppose, for example, that the expert prefers $R(O_{0.5})$ to $B(O_{0.5})$ in $CP_1(0.5)$ (we write $R(O_{0.5}) \succ B(O_{0.5})$) and $S^+(0.5)$ to $S^-(0.5)$ in $CP_2(0.5)$ ($S^+(0.5) \succ S^-(0.5)$). Assume that an expert's beliefs about the likelihood of the states (1)–(4) being realized can be represented with a set of subjective probabilities–p_1, p_2, p_3, and p_4 respectively. The only reason to prefer $R(O_{0.5})$ to $B(O_{0.5})$ is if you believe $p_1 + p_2 > p_3 + p_4$. The only reason to prefer $S^+(0.5)$ to $S^-(0.5)$ is if you believe $p_1 + p_3 > p_2 + p_4$. The only way to satisfy these two inequalities is if $p_1 > p_4$, i.e. your choices reveal that, if your beliefs are probabilistic, you believe state (1) is more likely than state (4).

Suppose now that you are offered a choice between $R(O_{0.5})$ and $S^-(0.5)$. Table 1 shows that these bets yield the same payoffs in states (2) and (3), so your beliefs about the relative likelihoods of these states should not affect whether you prefer $R(O_{0.5})$ or $S^-(0.5)$. However, $R(O_{0.5})$ pays out x in state (1), while $S^-(0.5)$ pays nothing, and vice versa in state (4). Your first two choices revealed you believed (1) is more likely than (4), so it follows that you should now prefer $R(O_{0.5})$ to $S^-(0.5)$ (similarly, we can show that you should prefer $S^+(0.5)$ to $B(O_{0.5})$). We thus have:

$$R(O_{0.5}) \succ B(O_{0.5}) \text{ and } S^+(0.5) \succ S^-(0.5) \Rightarrow R(O_{0.5}) \succ S^-(0.5) \text{ and } S^+(0.5) \succ B(O_{0.5}) \quad (1)$$

We can now select $CP_3(0.5)$ to be either of the betting pairs $[R(O_{0.5}), S^-(0.5)]$; $[S^+(0.5), B(O_{0.5})]$ – suppose we choose the former. If the expert chooses $S^-(0.5)$ over $R(O_{0.5})$ we are forced to infer that she believes $p_1 < p_4$, which contradicts her previous choices. In this case we conclude that there is no set of subjective probabilities (p_1, p_2, p_3, p_4) that can account for her preferences.

By similar reasoning, given any two initial choices over the two pairs of bets $CP_1(0.5)$ and $CP_2(0.5)$, we can predict the outcome of a third choice $CP_3(0.5)$ between a bet on Urn $O_{0.5}$ and a bet on the value of S, under the assumption that the expert possesses subjective probabilities over the state space. By offering a third choice, therefore, we are able to determine whether the expert's beliefs about S can be described by subjective probabilities. Table 2 contains an exhaustive list of all possible preferences over the first two betting pairs, and what we can infer from these preferences about preferences over the mixed betting pair. The mixed betting pair was contracted from the elements of the table, and SEU violations were registered if the preferences represented in the elements of the table were not observed.

Table 2: Preference possibilities, and inferred preferences over mixed bets, for the Climate Problem at the 50th percentile. The vertical axis contains all possible preferences for bets on the Objective Urn ($O_{0.5}$). The horizontal axis contains all possible preferences for bets on whether $S > S_{0.5}$. Elements of the table are the inferred preferences over mixed bets assuming the expert behaves according to Subjective Expected Utility theory.

	$S^+(0.5) \succ S^-(0.5)$	$S^+(0.5) \sim S^-(0.5)$	$S^+(0.5) \prec S^-(0.5)$
$R(O_{0.5}) \succ B(O_{0.5})$	$S^+(0.5) \succ B(O_{0.5})$	$S^+(0.5) \succ B(O_{0.5})$	$S^-(0.5) \succ B(O_{0.5})$
$R(O_{0.5}) \sim B(O_{0.5})$	$S^+(0.5) \succ B(O_{0.5})$	$S^+(0.5) \sim B(O_{0.5})$	$S^-(0.5) \succ B(O_{0.5})$
$R(O_{0.5}) \prec B(O_{0.5})$	$S^+(0.5) \succ R(O_{0.5})$	$S^+(0.5) \succ R(O_{0.5})$	$S^-(0.5) \succ R(O_{0.5})$

The Classic Ellsberg Problem was used to infer violations of SEU for the Climate Problem at $p = 0.5$, and for the Ellsberg Problem. In the latter case, outcomes $+$ and $-$ were replaced with the events R_e, B_e respectively, where R_e (B_e) denotes a draw of a red (blue) ball from Urn E, and the bets $S^+(0.5)$ and $S^-(0.5)$ were replaced with $R(E)$ and $B(E)$. Exactly the same reasoning applies to the Ellsberg Problem. In both cases x was given the hypothetical value of \$50.

Ellsberg Problem with unequal payoffs: In order to apply the structure of Ellsberg's Problem to bets on the value of S at the 5th and 95th percentile, it was necessary to modify the bets offered to the experts. To see why, suppose you have an urn that contains 100 balls, 95 of which are red, and 5 of which are blue, and one ball is drawn at random from the urn. You are unlikely to be indifferent between a bet that pays you \$100 if the drawn ball is red and a bet that pays \$100 if the drawn ball is blue. However, if you are risk neutral, you will be indifferent between a bet that pays out \$50 if the drawn ball is red, and a bet that pays out \$950 if the drawn ball is blue. The payoffs in these bets are calibrated to your beliefs about the composition of the urn, so indifference is guaranteed. Bets such as these with unequal payoffs were used in the Climate Problem at the 5th and 95th percentiles. Note that while the bets' payoffs are calibrated by using the convenient assumption of risk neutrality, our inferences about whether an experts' choices are consistent with SEU do *not* require us to assume that they are risk neutral. Inference about the existence (or lack thereof) of subjective probabilities for these bets is described below. For the sake of clarity, we focus on an example at $p = 0.05$:

Let R (B) be the event in which a red (blue) ball is drawn from Urn $O_{0.05}$, and $+$ ($-$) be the event that $S \geq S_{0.05}$ ($S < S_{0.05}$). We modified the payoffs in the classic Ellsberg Problem so that we now have the payoff structure depicted in Table 3, where $x =$ \$50, and $y =$ \$950 at $p = 0.05$. Suppose the expert prefers $R(O_{0.05})$ to $B(O_{0.05})$ in $CP_1(0.05)$, and $S^+(0.05)$ to $S^-(0.05)$ in $CP_2(0.05)$. We assume that the experts' choices may be represented by a subjective expected

Table 3: Modified Ellsberg Problem: Unequal payoffs

	(1) $(R,+)$	(2) $(R,-)$	(3) $(B,+)$	(4) $(B,-)$
$R(O_{0.05})$	x	x	0	0
$B(O_{0.05})$	0	0	y	y
$S^+(0.05)$	x	0	x	0
$S^-(0.05)$	0	y	0	y

utility functional $\tilde{V} = \sum_i U(x_i)p_i$, where x_i is the payoff in state i, $U(x_i)$ is the utility of payoff x_i, and p_i is the subjective probability of state i being realized, such that for two bets a and b:

$$a \succ b \Leftrightarrow \tilde{V}_a > \tilde{V}_b \tag{2}$$

We can then represent the expert's preferences with two inequalities:

$$U(x)[p_1+p_2] + U(0)[p_3+p_4] \quad > \quad U(0)[p_1+p_2] + U(y)[p_3+p_4] \tag{3}$$
$$U(x)[p_1+p_3] + U(0)[p_2+p_4] \quad > \quad U(0)[p_1+p_3] + U(y)[p_2+p_4] \tag{4}$$

Now add $[U(y)p_2 + U(0)p_3]$ to both sides of inequality 3, and add $[U(x)p_2 + U(0)p_3]$ to both sides of inequality 4. Rearranging yields:

$$U(x)[p_1+p_2] + U(0)[p_3+p_4] + [U(y)-U(0)][p_2-p_3] \quad > \quad U(0)[p_1+p_3] + U(y)[p_2+p_4]$$
$$U(x)[p_1+p_2] + U(0)[p_3+p_4] - [U(x)-U(0)][p_2-p_3] \quad > \quad U(0)[p_1+p_3] + U(y)[p_2+p_4]$$

If the utility function is increasing (i.e. high payoffs are preferred to low payoffs), as it surely must be, then we have that $\text{sgn}[U(y) - U(0)] = \text{sgn}[U(x) - U(0)]$. Thus the left hand side of these inequalities is always greater than the right hand side, whether we are adding the term $[U(y) - U(0)][p_2 - p_3]$ or subtracting it. Consequently, it must be the case that:

$$U(x)[p_1+p_2] + U(0)[p_3+p_4] \quad > \quad U(0)[p_1+p_3] + U(y)[p_2+p_4]$$
$$\Rightarrow R(O_{0.05}) \quad \succ \quad S^-(0.05)$$

By similar reasoning, we also know that $S^+(0.05) \succ B(O_{0.05})$. We have therefore found that

$$R(O_{0.05}) \succ B(O_{0.05}) \text{ and } S^+(0.05) \succ S^-(0.05) \Rightarrow R(O_{0.05}) \succ S^-(0.05) \text{ and } S^+(0.05) \succ B(O_{0.05}) \tag{5}$$

Thus, given the assumed choices in $CP_1(0.05)$ and $CP_2(0.05)$ in this example, the third betting pair $CP_3(0.05)$ is set to be either the pair $[R(O_{0.05}), S^-(0.05)]$ or $[S^+(0.05), B(O_{0.05})]$. If an expert makes choices on $CP_3(0.05)$ that contradict the predictions in equation (5), she violates SEU. Analogous reasoning allows us to construct relevant betting pairs in $CP_3(0.05)$ given any two initial choices in $CP_1(0.05)$ and $CP_2(0.05)$. Identical reasoning holds for bets at $p = 0.95$.

In analogy with Table 2, Table 4 represents all possible preference inferences for the betting pairs presented to experts at the 95th percentile. The corresponding table at the 5th percentile is similar.

Table 4: Preference possibilities, and inferred preferences over mixed bets, for the Climate Problem at the 95th percentile. The vertical axis contains all possible preferences for bets on the Objective Urn ($O_{0.95}$). The horizontal axis contains all possible preferences for bets on whether $S > S_{0.95}$. Elements of the table are the inferred preferences over mixed bets assuming the expert behaves according to Subjective Expected Utility theory.

	$S^+(0.95) \succ S^-(0.95)$	$S^+(0.95) \sim S^-(0.95)$	$S^+(0.95) \prec S^-(0.95)$
$R(O_{0.95}) \succ B(O_{0.95})$	$S^+(0.95) \succ B(O_{0.95})$	$S^+(0.95) \succ B(O_{0.95})$	$S^-(0.95) \succ B(O_{0.95})$
$R(O_{0.95}) \sim B(O_{0.95})$	$S^+(0.95) \succ R(O_{0.95})$	$S^+(0.95) \sim R(O_{0.95})$	$S^-(0.95) \succ R(O_{0.95})$
$R(O_{0.95}) \prec B(O_{0.95})$	$S^+(0.95) \succ R(O_{0.95})$	$S^+(0.95) \succ R(O_{0.95})$	$S^-(0.95) \succ R(O_{0.95})$

Note: In the **Classic Ellsberg Problem** we are able to predict choices consistent with the existence of subjective probabilities without any auxiliary assumptions. Hence, choices that contradict these predictions imply that no probabilities exist that account for the observed choices. We are able to rule out arbitrary probabilistically sophisticated [2] preference representations, and not only the SEU preference representation, in this case. In the **Ellsberg Problem with unequal payoffs**, our argument assumed that preferences could be represented by the function $\tilde{V} = \sum_i U(x_i)p_i$ (the SEU representation). In fact, our reasoning in this case applies without modification to a more general preference representation which ranks lotteries via any linear combination of subjective expected utilities:

$$V = \sum_i \sum_j U_j(x_i) f_{ij}(p_i) \tag{6}$$

It is easy to verify that our predictions in equations 5 are valid for this larger class of both expected utility maximisers and probabilistically sophisticated non-expected utility maximisers [2]. Thus, as long as we can represent a expert's preferences with a function of the form in equation 6, choices contrary to our predictions in equation 5 imply that no probabilities exist that would account for the observed choices. This is a slightly weaker statement than we are able to make for the Classic Ellsberg Problem, but given the generality of the functional form (6) it is reasonable to interpret SEU violations as a very strong indicator that the expert's beliefs cannot be represented with probabilities.

2 Supplementary discussion

We emphasise that the design of our choice experiment is deliberately conservative with respect to the non-existence of subjective probabilities, for the following reasons:

1. Experts were asked to complete a conventional expert elicitation exercise before seeing the betting questions. The elicitation exercise asked the climate scientists to represent their beliefs about climate sensitivity in terms of a probability distribution, thus priming them to think about their uncertainty in these terms. In addition, as noted above, the Ellsberg

Problem was presented at the very end of the survey, to avoid priming the subjects to think in terms of ambiguity when responding to the choice tasks.

2. We presented the relevant sets of bets (1 pair of bets on an urn of known composition, 1 pair of bets on S, 1 mixed pair of bets, at each percentile of S) in immediate succession, rather than scattering them across the survey. The intention was to make the experiment as transparent as possible, so that experts could easily see its structure and make choices consistent with SEU.

3. We selected a sample of climate science experts to answer betting questions about their domain of expertise. The experimental literature suggests that, contrary to the usual effect of ambiguity aversion, people often prefer to bet on their ambiguous beliefs in situations where they feel especially competent or knowledgeable [4]. Our experimental design would register this behaviour as consistent with the existence of subjective probabilities.

Statistical analysis

This section provides details of our statistical argument for the presence of ambiguity about S in our sample of experts. Using the logic described in the supplementary methods, it is possible to infer whether experts violate SEU for each set of bets offered to them. Our analysis does not treat a violation of SEU for bets on the value of S by a single individual as unequivocal evidence that she has ambiguous beliefs about S. Idiosyncratic factors (e.g. reasoning errors, misreading of the survey questions, imperfect calibration of the payoffs in the bets to the expert's risk preferences) may cause individuals to violate SEU (or not) for reasons other than ambiguity. Our argument thus uses statistical analysis to demonstrate that there are ambiguous beliefs about S in the sample.

In each of the analyses that follow we represent the relevant data in 2×2 contingency tables, and use Barnard's exact test [5] to test for independence between the rows of the tables. A common alternative to Barnard's test, Fisher's exact test, is less powerful for 2×2 contingency tables, producing overly conservative P-values [5, 6].

As discussed in the paper, if an expert is ambiguity averse on the Ellsberg Problem, she should also be more likely to exhibit ambiguity aversion (and hence violate SEU) on the Climate Problem if she has ambiguous knowledge about S. Thus, if there is ambiguity in the Climate Problem, we would expect SEU violations in bets on S to be relatively more common among experts who violated SEU on the Ellsberg Problem. In the absence of this causal link, one would expect SEU violations on the Ellsberg Problem and on the Climate Problem to be independent events.

Table 5 shows that violation of SEU on at least one of the sets of bets offered at the 5th, 50th and 95th percentiles of the expert's elicited distribution for S in the Climate Problem are relatively more common among those that violate SEU on the Ellsberg Problem (33% versus 10%). A Barnard test rejects the hypothesis that SEU violations on S are independent of SEU violations on the Ellsberg Problem at the 10% level ($P = 0.083$). This suggests that ambiguity about S drives SEU violations in the Climate Problem.

Theoretical arguments [7] suggest that the upper tail of the distribution for S is less constrained by the instrumental record than the rest of the distribution. This provides a further opportunity to test our hypothesis that ambiguous beliefs are causing the SEU violations in bets on the value of S.

Table 5: Contingency table for SEU violations on the Ellsberg Problem (EP) and Climate Problems (CP) ($N = 29$)

	Violation of SEU on CP	No violation of SEU on CP
Violation of SEU on EP	3	6
No violation of SEU on EP	2	18

We consider the variants of the Climate Problem at each of the three percentiles of S separately. If ambiguous beliefs are causing the SEU violations, we would expect it to be more difficult to reject the hypothesis that SEU violations on the Climate and the Ellsberg Problems are independent at higher percentiles. The results are reported in Table 6. The pattern of P-values is consistent with the hypothesis that climate scientists have ambiguous beliefs about climate sensitivity, which are largely driven by ambiguity at large values of S.

Table 6: P-values from Barnard tests of independence of SEU violation on S from SEU violations on the Ellsberg Problem at the 5th, 50th, and 95th percentiles ($N = 29$)

	P-value
5th percentile	0.501
50th percentile	0.138
95th percentile	0.045

The data in Table 7 illustrate the distinction between risk and ambiguity, in that SEU violations are not disproportionately common for experts that report a greater 5-95% spread for S, i.e. ambiguous beliefs about climate sensitivity are independent of the width of the expert's elicited probability distribution for S ($P = 0.574$).

Table 7: Risk versus Ambiguity in the Climate Problem ($N = 29$)

	Violation of SEU on CP	No violation of SEU on CP
Above median 5-95% spread	3	12
Below median 5-95% spread	2	12

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